TABLE I

AMPLITUDE	POSITION
$+1$	α
$(-1)^{i+1} C_i^j C_j^k\ k_1^{(i-j+k)} . k_2^{(1+j)} . k_3^{(i-k)}$	$-\alpha+2a_1+2b_1(i-j+k)+2b_2(i-k)$
$(-1)^{i+1} C_i^j C_j^k\ k_1^{(i-j+k)} . k_2^{j} . k_3^{(1+i-k)}$	$-\alpha+2a_2+2b_1(i-j+k)+2b_2(i-k)$
$(-1)^{i+1} C_i^j C_j^k\ k_1^{(1+i-j+k)} . k_2^{(1+j)} . k_3^{(i-k)}$	$+\alpha+2b_1(1+i-j+k)+2b_2(i-k)$
$(-1)^{i+1} C_i^j C_j^k\ k_1^{(1+i-j+k)} . k_2^{j} . k_3^{(1+i-k)}$	$+\alpha+2b_1(1+i-j+k)+2b_2(1+i-k)$
k_1	$-\alpha+2a_0$
$(-1)^{i+1} C_i^j C_j^k\ k_1^{(1+i-j+k)} . k_2^{(1+j)} . k_3^{(i-k)}$	$+\alpha-2b_1(1+i-j+k)-2b_2(i-k)$
$(-1)^{i+1} C_i^j C_j^k\ k_1^{(2+i-j+k)} . k_2^{(1+j)} . k_3^{(i-k)}$	$-\alpha+2a_0-2b_1(1+i-j+k)-2b_2(i-k)$
$(-1)^{i+1} C_i^j C_j^k\ k_1^{(1+i-j+k)} . k_2^{j} . k_3^{(1+i-k)}$	$+\alpha-2b_1(1+i-j+k)-2b_2(1+i-k)$
$(-1)^{i+1} C_i^j C_j^k\ k_1^{(2+i-j+k)} . k_2^{j} . k_3^{(1+i-k)}$	$-\alpha+2a_0-2b_1(1+i-j+k)-2b_2(1+i-k)$

$i = 0, \infty$

$j = 0, i$

$k = 0, j$

$$C_i^j = \frac{i!}{j!\,(i-j)!}$$

$$k_n = \frac{\varepsilon_n - \varepsilon_{n-1}}{\varepsilon_n + \varepsilon_{n-1}}, \quad n = 1, 2, 3.$$

TABLE II

AMPLITUDE	POSITION
$+1$	h
$+k_1$	$-h$
$+k_1^{2i+1}$	$-h-2hi$
$+k_1^{2i}$	$+h-2hi$
$+k_1^{2i}$	$+h+2hi$
$+k_1^{2i-1}$	$-h+2hi$

imposed upon the general configuration (Fig. 2) in order to take into account the conductors:

$$\varepsilon_0 \longrightarrow \infty \qquad \varepsilon_3 \longrightarrow \infty.$$

This implies that $k_1 = 1$ and $k_3 = 1$; the expressions (16) given by Coen can then be found.

Without having to go into details, we would like to mention the practical problems which are solved (in using the results shown in Table I).

In a three-dimensional case, we studied the influence of an upper ground plane and a medium of several dielectrics on the elements of an equivalent circuit for discontinuities. The curves obtained recover exactly those given in (3).

In the two-dimensional case, this generalized method of images allows the propagation in structures shown in Fig. 1(b)–(d) to be treated [4]. Moreover, for coplanar conductors on a dielectric substrate, the formulas of Table I can be considerably simplified.

As an hypothesis, let $\varepsilon_2 = \varepsilon_3$ ($k_3 = 0$). The configuration [Fig. 1(e)] can then be easily treated. The amplitudes of charges decrease extremely rapidly as shown in Table II.

The geometrical configuration [Fig. 1(d)] allows a configuration identical to the preceding one to be treated but sandwiched.

We can conclude that depending upon the chosen integral representation, this method allows the two- or three-dimensional case to be treated by avoiding the singularity of the point source. This method is very interesting in solving open or semiopen problems. Knowing Green's function in a four dielectric medium, a general program valid for most transmission lines used in hyperfrequency microelectronics has been realized.

References

[1] P. Silvester, "TEM wave properties of microstrip transmission lines," *Proc. Inst. Elec. Eng.*, vol. 115, pp. 43–48, Jan. 1968.

[2] J. A. Weiss and T. G. Bryant, "Dielectric Green's function for parameters of microstrip," *Electron. Lett.*, vol. 6, pp. 462–464, July 23 1970.

[3] P. Benedek and P. Silvester, "Equivalent capacitances for microstrip gaps and strips," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 729–733, Nov. 1972.

[4] E. Yamashita, "Variational method for the analysis of microstrip-like transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 529–535, Aug. 1968.

[5] Shimon Coen, "A note of Green's function for microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 591–593, July 1975.

[6] G. N. Watson, "A treatise on the theory of Bessel's functions," Cambridge at the University Press, 1952.

[7] E. Durand, Electrostatique, Tome III Masson Editeur, Paris 1966.

The Locus of Points of Constant VSWR When Renormalized to a Different Characteristic Impedance

J. A. G. MALHERBE

***Abstract*—The locus of points of constant VSWR with respect to an impedance Z_1, when measured with equipment of characteristic impedance Z_0, other than Z_1, is found to be a circle that is easily constructed on a Smith chart. The use of a transparent overlay with various such circles converts any network analyzer with a Smith chart display to read swept VSWR values to the new impedance.**

In cases where the VSWR of a device is specified with respect to an impedance Z_1 other than the characteristic impedance of the measuring equipment Z_0 (50 Ω), as is often the case with TV antenna equipment or cables, the VSWR has to be calculated from complex impedance measurements on a point-by-point basis. Thus the advantages of having a network analyzer with a sweep oscillator and Smith chart display are lost because the normal constant VSWR circles would refer to Z_0 rather than Z_1. This letter shows that the locus of points of constant VSWR to Z_1 are again circles, and that they can be very easily constructed, enabling the use of a transparent overlay and thus regaining the original network analyzer advantages.

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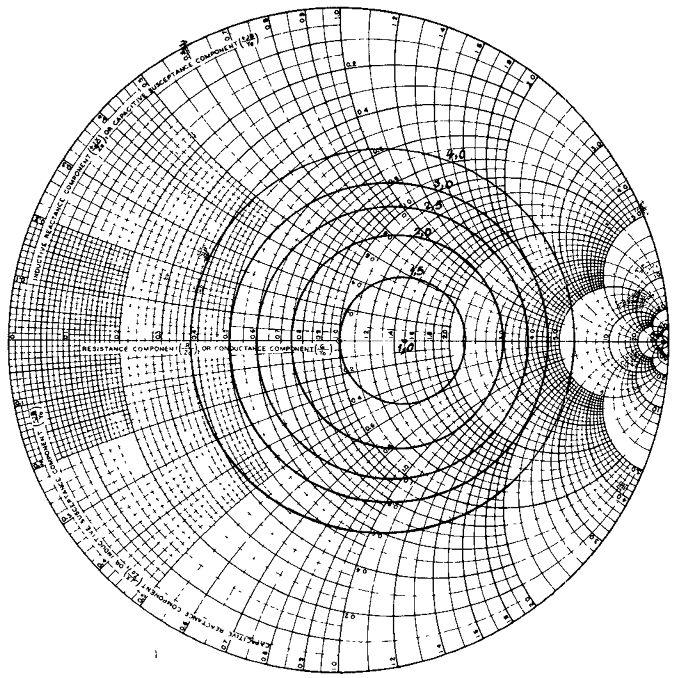

Fig. 1. Smith chart with circles of constant VSWR for an impedance of $z = 1.5$.

Consider an impedance $R + jX$, which has a constant VSWR S, with respect to an impedance $Z_1 = zZ_0$. The reflection coefficient is given by

$$\rho = \frac{R + jX - Z_1}{R + jX + Z_1}$$

therefore,

$$|\rho|^2 = \frac{(R - Z_1)^2 + X^2}{(R + Z_1)^2 + X^2} = \left(\frac{S - 1}{S + 1}\right)^2. \tag{1}$$

For any $\rho = x + jy$ to another impedance Z_0,

$$R + jX = Z_0 \frac{1 + \rho}{1 - \rho} = Z_0 \frac{1 + x + jy}{1 - x - jy}. \tag{2}$$

Separating the real and imaginary equations in (2), and substituting R and X as functions of x, y, and Z_0 into (1) yields, after considerable manipulation,

$$(x - x_0)^2 + y^2 = a^2 \tag{3}$$

in which

$$x_0 = \frac{S(z^2 - 1)}{(Sz + 1)(z + S)} \tag{4}$$

$$a = \frac{z(S^2 - 1)}{(Sz + 1)(z + S)}. \tag{5}$$

Equations (3)–(5) are seen to describe a family of circles with center points on the Smith chart real axis and offset a distance x_0 from the origin, and of radius a. These circles intersect the real-part axis of the Smith chart at points

$$r_1 = zS \qquad r_2 = z/S.$$

Thus, given *complex* values on the Z_0 chart [values of $S(Z_0)$ alone will not suffice], $S(Z_1)$ can be read off directly from the constructed circles, shown in Fig. 1 for $z = 1.5$ and various values of S.

Correction to "Efficient Calculation of Exact Group Delay Sensitivities"

J. W. BANDLER, M. R. M. RIZK, AND H. TROMP

In the above paper,[1] on page 190, the following corrections should be made.

Equation (15) should read

$$\begin{bmatrix} I_{aj}' \\ V_{bj}' \end{bmatrix} = - \begin{bmatrix} I_{aj}'^{s} \\ V_{bj}'^{s} \end{bmatrix} + \begin{bmatrix} Y_j & A_j \\ M_j & Z_j \end{bmatrix} \begin{bmatrix} V_{aj}' \\ I_{bj}' \end{bmatrix} \tag{15}$$

to match the sign convention given in Fig. 1. Consequently, (17) is

$$\begin{bmatrix} \dfrac{\partial V_{aj}}{\partial \omega} \\ \dfrac{\partial I_{bj}}{\partial \omega} \end{bmatrix} = - \begin{bmatrix} V_{aj}' \\ I_{bj}' \end{bmatrix}. \tag{17}$$

It should also be noted that in (13), the G_i' corresponds to the sensitivity component given by Bandler and Seviora,[2] but with opposite sign.

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[1] J. W. Bandler, M. R. M. Rizk, and H. Tromp, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 188–194, Apr. 1976.
[2] J. W. Bandler and R. E. Seviora, "Current trends in network optimization," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 1159–1170, Dec. 1970.